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Review

The ideas of dual description of quantum chromodynamics

H Nandan* and H C Chandola[1]

Centre for Theoretical Studies, Indian Institute of Technology, Kharagpur-721 302, West Bengal, India

[1]Department of Physics, Kumaun University, Nainital-263 001, Uttarakhand, India

E-mail : hnandan@cts.iitkgp.ernet.in

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Abstract : Starting with the landmark developments those lead to the standard model (SM) of modern elementary particle physics, the notions of quantum chromodynamics (QCD) are discussed. The basic aspects of the necessary ingredients (*viz.* monopoles and dyons) along with their inherent presence in QCD to construct a dual chromodynamic vacuum are reviewed. The gauge-theoretical formulation of QCD and its analogy with superconductivity is addressed in view of the magnetic condensation leading to the dual Meissner effect in QCD vacuum. The colour confinement mechanism in view of the flux tube model is also addressed in brief. Finally, the circumstances under which the dual superconducting behaviour of QCD vacuum sets in, are summarised.

Keywords : Monopoles, dyons, dual QCD, confinement, flux tube and Regge trajectories.

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Plan of Article

*Corresponding Author

1. Introduction

The current idea about the fundamental building blocks began to show up with the construction of the multitude of hadrons (*viz.* protons and neutrons) into the irreducible representations of the unitary symmetry groups through all the possible combinations of constituent particles known as quarks [1,2]. The six flavours namely *u* (up), *d* (down), *s* (strange), *c* (charm), *b* (bottom) and *t* (top) [1–7] are regarded as the basic building blocks of matter in addition to the leptons (*viz.* electrons). Besides the electrons, there are muon (μ) and tau (τ) leptons along with their associated neutrinos (electron-, muon- and tau-neutrinos) which are electrically neutral and massless and represented by ν_e, ν_μ, and τ_ν respectively. The modern physics deals with the four fundamental interactions (gravitation, electromagnetic, weak and strong) among these elementary particles which are usually based on the principles of modern gauge theory and the range of these interactions is as variable as their strength [8]. Further, with the increasingly supporting evidences for the coloured quarks and strong interactions among them, the gauge field theory of strong interactions [9] has also emerged as the $SU(3)_C$ colour gauge theory, commonly known as the Quantum Chromodynamics (QCD), where the quarks interact *via* the exchange of a set of self-interacting gluons [8,10]. Further, a putative Grand Unified Theory (GUT) to combine the electro-weak (EW) and strong interactions leads to the SM of elementary particle physics which has blossomed into a vast subject, even though such unified description has not become truly apparent because of the large difference in the coupling strengths of the interactions [8]. The different properties of the fundamental particles and their interactions with the recent developments are time-to-time updated by the Particle Data Group [11].

The dynamical evidence for the existence of the quarks inside the observed hadrons has been gathered in a definite way through the scaling in deep-inelastic experiments used to probe the structure of the nucleus [12]. The typical confining inter-quark force which among quarks in the interiors of hadrons appears as a special feature of strong interactions and has its close links with the additional colour degrees of freedom of quarks [13]. In spite of a number of studies of coloured quark dynamics, a completely satisfactory mechanism of quark (colour) confinement is still lacking which may be derived through the first principles of QCD. The intractability of confinement problem in QCD has inspired various models and most prominent among them are the string and bag models [14,15], but the detailed investigations are still needed to have a complete description of the confinement mechanism. In this connection, it has been found that the unification of the strong interactions with other fundamental interactions, using the most popular non-Abelian gauge formulations which undergo spontaneous symmetry breaking, in general, leads to the appearance of the topological solitonic solutions [16] which are of finite energy, topologically stable localized configurations with the quantised magnetic charges [17]. The current interest in the possibility of magnetic monopoles resurfaced with the advent of '*t* Hooft-Polyakov monopoles in the non-Abelian gauge theory [18,19]. The quantum mechanical excitations of such fundamental monopoles further include dyons (objects carrying both the electric and magnetic charges [20]) which arise as a result of the excitation of electric degrees of freedom of the monopoles [21]. The electric charge generalisation of monopoles with finite energy [22] and the prediction of Grand Unified Theories (GUTs) about the non-zero charge states associated with monopoles [23] have further boosted the study of dyons [24]. The existence of such topological objects has, in recent times, gathered enormous interest mainly in connection with the various issues related to the large-scale structure of QCD which can perhaps be derived from the magnetically condensed state of QCD vacuum [24]. In such QCD vacuum, the colour electric flux lines between quarks are aligned into a tube [25–27] in a way analogous to the magnetic flux confinement in terms of the vortex lines in superconductivity [28]. This apparently rigorous connection between QCD and superconductor scenario is also clearly evident in view of the '*t* Hooft's Abelian gauge fixing technique [29] which reduces QCD into Abelian gauge theory where the coloured monopoles and dyons appear as topological objects and provide a theoretical basis for the magnetic condensation in QCD vacuum. Hence a dual QCD where the dual potentials are the natural variables to describe the large-distance properties of QCD vacuum which has also been widely investigated by using the modern lattice gauge theories for QCD [30,31]. In view of the essence of monopoles and dyons in constructing dual QCD, some of the basic aspects associated to them as well as their inherent presence in QCD are reviewed in the forthcoming sections.

2. Basic aspects of monopoles and dyons

The concept of monopoles, in recent years, has played key role in understanding various seemingly divergent problems related to the particle physics at both the classical and quantum level of its description. In the classical electrodynamics, the existence of the isolated electric charges and the impossibility to separate the magnetic poles is a well-known fundamental difference between the electricity and magnetism. Such difference may clearly be seen in terms of the Maxwell's equations of classical electrodynamics where the source of the magnetic field remains absent. However, the existence of a source of the magnetic field may allow to write the Maxwell's equations in a dual symmetric form. This pleasant symmetry at the level of electric and magnetic fields then leads to the existence of Dirac monopole [17] and explains the origin of the electric charge quantisation.

2.1. The Dirac monopole :

The fundamental physical laws of electrodynamics are generally expressed in terms of the manifestly covariant set of Maxwell's equations which have perfect built-in-dual structure in vacuum. However, since they lose their dual symmetry in presence of charged matter, it led Dirac [17] to introduce the quantised singularities in electromagnetic field to preserve the dual symmetric structure of electrodynamics which ultimately demonstrated that the existence of a single monopole can explain the quantisation of the electric charge of the universe. The Maxwell's equations in their covariant form [32] may be written in terms of the general electromagnetic field strength tensor ($F_{\mu\nu}$) as,

$$\partial^{\nu} F_{\mu\nu} = -j_{\mu}, \quad \partial^{\nu} \tilde{F}_{\mu\nu} = 0 . \tag{1}$$

where, $F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}$, $\tilde{F}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\sigma\rho} F^{\sigma\rho}$ is the dual electromagnetic field tensor and $j^{\mu} = (e, \boldsymbol{j})$ is the electric four current. The set of eqs. (1) is completely dual symmetric in vacuum (*i.e.* $j_{\mu} = 0$) under the following set of duality transformations,

$$\begin{pmatrix} F_{\mu\nu} \\ \tilde{F}_{\mu\nu} \end{pmatrix} \to \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} F_{\mu\nu} \\ \tilde{F}_{\mu\nu} \end{pmatrix} \to R(\theta) \begin{pmatrix} F_{\mu\nu} \\ \tilde{F}_{\mu\nu} \end{pmatrix} . \tag{2}$$

In particular, for $\theta = (\pi/2)$, the set of transformations given by eq. (2) corresponds to the complete interchange of electricity and magnetism (*i.e.* $\boldsymbol{E} \to \boldsymbol{B}$, $\boldsymbol{B} \to -\boldsymbol{E}$). The duality symmetry is spoiled in the presence of the electric source ($j_{\mu} \neq 0$) may, however, be restored by introducing an additional magnetic four current vector $k^{\mu} = (\sigma, \boldsymbol{k})$ in the set of eqs. (1) as $\partial^{\nu} \tilde{F}_{\mu\nu} = -k_{\mu}$. With the introduction of k_{μ}, the set of the duality transformations given by eq. (2) is then supplemented by another set of transformations $(j_{\mu}, k_{\mu})^{T} \to R(\theta)(j_{\mu}, k_{\mu})^{T}$. The introduction of magnetic current (k_{μ}) naturally requires the existence of magnetically charged particles (*i.e.* the magnetic monopoles) and in analogy to the electric current, the magnetic current due to the magnetic charges of the strength g_i is then given by,

$$k_{\mu}(x) = \sum_{i} g_i \int dx_{\mu}^{i} \delta^{4}(x - x_i). \tag{3}$$

The magnetic field due to a monopole of strength g is then given by $\boldsymbol{B} = \left(g/4\pi r^2\right) \hat{r}$, where, $\hat{r}$ is the unit vector in the radial direction [32]. The motion of an electric charge q in the magnetic field of a monopole is, therefore, given by the equation of motion in the following form,

$$m\ddot{x}_i = q \in_{ijk} \dot{x}^j B^k. \tag{4}$$

The time evolution of its angular momentum then suggests to identify the associated conserved angular momentum in the following form,

$$J_k = m \in_{ijk} x^i \dot{x}^j - \frac{qg}{4\pi} \hat{x}_k. \tag{5}$$

The second term in eq. (5) appears due to the poynting vector $\boldsymbol{E} \times \boldsymbol{B}$ in the overlapping region of the fields of the electric and magnetic charges. With the quantisation of the theory, the corresponding angular momentum components are expected to satisfy the usual commutation relations which implies that the eigen values of J_k are half integers and, in turn, leads to the celebrated Dirac quantisation condition given by $qg/4\pi = n/2$, where, n is an integer. It indicates that the electric charge is quantised in nature. It is also worth mentioning that the quantisation condition can also be heuristically derived by considering the angular momentum of dyon [33]. However, in the presence of the magnetic monopole, the gauge (vector) potential $\boldsymbol{A}$ can not exist everywhere because $\tilde{F}_{\mu\nu}$ now satisfies $\partial^\nu \tilde{F}_{\mu\nu} = -k_\mu$. The Gauss law $g = \int \boldsymbol{B} \cdot d\boldsymbol{S}$ for the magnetic charge then indicates that $\boldsymbol{B}$ can not be written as the curl of the vector potential everywhere. However, one can define the vector potential $\boldsymbol{A}$ in such a way that $\boldsymbol{B} = \nabla \times \boldsymbol{A}$ everywhere except on the line joining from the origin to infinity which leads to the existence of Dirac string. For this purpose, the magnetic field may now consider due to an infinitely long thin solenoid along the negative z-axis with its positive pole of strength at the origin [32]. The vector potential which is singular through out the entire negative *z-axis* may then be expressed with the conventional definition of polar and azimuthal angles in the following form,

$$\boldsymbol{A} = \frac{g}{4\pi r}\left(\frac{1-\cos\theta}{\sin\theta}\right)\hat{\phi}. \tag{6}$$

The direction of Dirac string can be chosen to be along any direction by a suitable choice of coordinates and need not to be straight, but must be continuous. The monopole field may therefore be represented by the vector potential together with the Dirac string. The Dirac string is not physically observable as it does not give rise to the Aharonov-Bhom effect [34] and it is, therefore, obviously desirable to get rid of such unphysical object. A pictorial representation of the Dirac monopole with a string is presented in Figure 1. In this connection, Wu-Yang interpretation [35] suggested an elegant formulation to eliminate the Dirac string associated to monopoles by the partition of the space surrounding the magnetic monopole of strength $g \neq 0$ in to two different but overlapping regions called *patches.* In each of such regions, the vector potential due to the monopole is non-singular, but related by a transition gauge

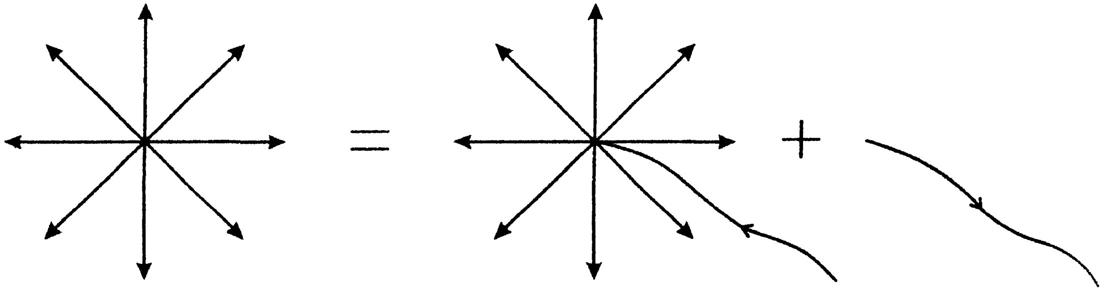


Figure 1. Dirac monopole with a string.

transformation and leads to the Dirac quantisation condition in a geometric way. This approach is, in essence, a fibre-bundle formulation of a monopole [36] where the base space (R^3 minus the origin : R^3 – {0}) is essentially the sphere S^2 (the sphere surrounding the monopole) provides the fruitful topological insights those are accompanied by a magnetic monopole. Moreover, in recent years, the 't Hooft-Polyakov monopoles [18,19] along with their extension to dyons [21] in various unified non-Abelian gauge theories [37] have further motivated physicists for their eventual applications to solve various problems in particle physics.

2.2. The 't Hooft-Polyakov monopole :

The simplest example of a non-Abelian gauge theory having monopole solution is the Georgi-Glashow model with the $SU(2)$ symmetry group and Higgs triplet (ϕ) [38]. The Lagrangian density with only the bosonic part may then be considered in the following form,

$$L = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \frac{1}{2} (D_\mu \phi)^a \cdot (D^\mu \phi)^a - V(\phi), \tag{7}$$

where, the gauge field strength tensor, the covariant derivative and potential are defined respectively in the form given below,

$$G^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g \in^{abc} W^b_\mu W^c_\nu, \tag{8}$$

$$\left(D_\mu \phi\right)^a = \partial_\mu \phi^a - g \in^{abc} W^b_\mu \phi^c, \tag{9}$$

$$V(\phi) = \frac{\lambda}{4}\left(\phi \cdot \phi - \phi_0^2\right)^2, \tag{10}$$

where a, b, c = 1, 2, 3. The field equations corresponding to the Lagrangian (7) are then derived in the following form,

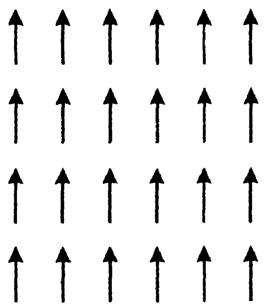
$$\left(D_\mu D^\mu \phi\right)^a = -\lambda \phi^a \left(\phi \cdot \phi - \phi_0^2\right), \tag{11}$$

$$\left(D_\mu G^{\mu\nu}\right)^a = -g \in^{abc} \left(D^\nu \phi\right)^b \phi^c. \tag{12}$$

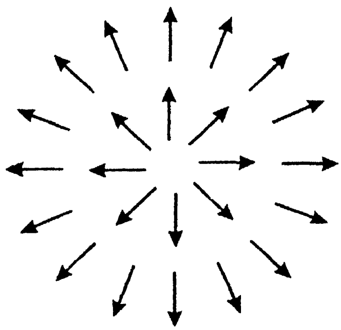
The values of ϕ which minimise the potential $V(\phi)$ in eq. (10) are given by,

$$M_0 \equiv \left\{\phi = \eta, \eta^2 = \phi_0^2 : V(\eta) = 0\right\}, \tag{13}$$

where, M_0 consists of the points on a sphere in a 3-dimensional internal symmetry space. Let us use, the ground state configuration as φ = (0, 0, ϕ_0) which remains invariant under the rotation around the third-axis. The pattern of spontaneous symmetry breaking path now becomes $SU(2) \to U(1)$, where the $U(1)$ symmetry can be assigned to the electromagnetic interaction which corresponds to the massless gauge field (photon). In order to find the finite energy solution, $\phi(\boldsymbol{r})$ must approach to its value in M_0 with the variation in spatial coordinate as $r \to \infty$. The possible directions in which r tends to infinity is determined by the unit vector as $S^2 : \{\hat{\boldsymbol{r}} : \hat{\boldsymbol{r}}^2 = 1\}$ which has the same topology as the set M_0. It is, therefore, evident that each point in the sphere (S^2) may be mapped to the corresponding point in S^2 of M_0 The mapping $S^2 \to S^2$ then leads to a non-trivial topology defined by $\phi_i^\infty = \eta_i = \phi_0 \hat{r}_i$ which is topologically stable as it can not be deformed continuously into the mapping for the vacuum configuration where the whole S^2 is mapped to a point and known as 'hedgehog' mapping. This mapping leads to the monopole-like topological solitons and the remarkable difference between the vacuum and soliton state corresponding to the 'hedgehog mapping' is exhibited in Figures 2(a) and (b) respectively. Further, in order



(a)



(b)

Figure 2. Schematic configuration for the complex scalar field (ϕ) for . (a) vacuum and (b) soliton (monopole).

to obtain the lowest energy configurations, the explicit solutions corresponding to those with the maximal symmetry, one can use the following set of ansatz,

$$\phi^a = \frac{r^a}{gr^2} H(\phi_0 gr), \; W_i^a = - \epsilon_{aij} \frac{r^j}{gr^2} \left\{1 - K(\phi_0 gr)\right\}, \; W_0^a = 0, \tag{14}$$

where, H and K are the dimensionless functions For the ansatz given by eq. (14), the energy of the system is given by,

$$E = \frac{4\pi\phi_0}{g}\int_0^{\infty}\frac{d\xi}{\xi^2}\left[\xi^2\left(\frac{dK}{d\xi}\right)^2 + \frac{1}{2}\left(\xi\frac{dH}{d\xi} - H\right)^2 + \frac{1}{2}\left(K^2 - 1\right)^2 + K^2H^2 + \frac{\lambda}{4g^2}\left(H^2 - \xi^2\right)^2\right] \tag{15}$$

where, $\xi = \phi_0 gr$ The conditions for energy to be stationary with the variations of H and K are given by,

$$\xi^2K'' - KH^2 - K(K^2 - 1) = 0, \tag{16}$$

$$\xi^2H'' - 2K^2H - \lambda g^{-2}H(H^2 - \xi^2) = 0, \tag{17}$$

where, the primes denote the differentiation with respect to ξ. In view of the asymptotic conditions and in order to have the convergent integral for the energy given by eq (15) the asymptotic boundary conditions (*i e* $\xi \to \infty$) for $H(\xi)$ and $K(\xi)$ are given as $H(\xi) \sim \xi$ and $K(\xi) \to 0$, respectively With the asymptotic behaviour of $K(\xi)$, at large distances,

$$G_{ij}^a \sim \frac{1}{gr^4}\in_{ijk} r^a r^k \sim \frac{1}{\phi_0 gr^3}\in_{ijk} r^k\phi^a, \tag{18}$$

the magnetic field in the asymptotic limit is given as $\boldsymbol{B} \sim -(1/gr^3)\boldsymbol{r}$ which behaves asymptotically in the same way as the Dirac monopole with the magnetic charge $g_m = 4\pi/g$ Such topologically meaningful stable finite energy state solution is called '*t* Hooft-Polyakov monopole. The monopoles arising in the non-Abelian gauge theories, in contrast to the Dirac monopoles, have a well defined internal structure The electric excitations of such monopoles are discussed in the next section

2.3. The Julia-Zee dyon :

Julia and Zee [21] generalised the '*t* Hooft-Polyakov monopole in the sense that the finite energy state solutions also carry the electric charge along with the usual magnetic charge [39,40]. For this purpose, the ansatz for $SU(2)$ fields given by eq. (14) are supplemented with that given below,

$$W_0^a = \frac{r^a}{gr^2}J\left(\phi_0 gr\right), \tag{19}$$

where, $J \to 0$ as $r \to 0$. With the eq (19), the field equations given by (16) and (17) are also modified as well as one obtains a new field equation in the the following form,

$$\xi^2K'' - K\left(K^2 - J^2 + H^2 - 1\right) = 0, \tag{20}$$

$$\xi^2H'' - 2K^2H - \lambda g^{-2}H\left(H^2 - \xi^2\right) = 0, \tag{21}$$

$$\xi^2 J'' - 2JK^2 = 0. \tag{22}$$

The eqs. (20–22) in the Bohr-Prasad-Sommerfeld (BPS) limit ($\lambda \to 0$) for dyons can be solved exactly [41,42] and the electric charge associated with these solutions is given as $Q_e = \int dS^i G_{0i}$. The 't Hooft Polyakov monopole therefore acquires the electric degree of freedom and becomes a dyon There also exists a Bogomol'nyi equation which holds in the limit of vanishing λ and is given as,

$$B_i^a \pm D_i \phi^a = 0, \tag{23}$$

where, B_i denotes the magnetic field. However, the electric charge excitation of such a monopole vanishes with $J = 0$ and $W_0^a = 0$ (*i.e.* vanishing of temporal degree of freedom of the gauge field). Moreover, the existence of such topological objects is not only advocated as a general consequence of the GUTs, but are also the objects of interest in cosmology and astrophysics which provides the possibility of the monopole abundance in nature [43]. Furthermore, the supersymmetric and lattice sectors of QCD also lead to the existence of these objects with all the necessary features [44,45] The monopoles and dyons are essential degrees of freedom (DOF) for dual QCD and their appearance in QCD as an Abelian theory is also discussed in Subsection 3 4 and before dealing with it, we have discussed some basics of QCD in the forthcoming sections.

3. Fundamentals of strong interactions

Among the four fundamental interactions of nature, the only interaction responsible for the nuclear stability is known as the strong force which has the highest strength. The three remarkable facts concerning strong nuclear interactions are that they are of short range, spin dependent and independent of the electric charges of the constituents of nucleus. Some important aspects of such interactions with their gauge-theoretical formulation and dual description of QCD vacuum have been discussed in the Sections 3 1–3.5.

3.1. Basic formalism of QCD with quarks and gluons .

The naive quark model points out that all the hadrons are built out of fractionally charged spin −1/2 quarks which form the fundamental representation of unitary $SU(3)$ (or ($SU(6)$)) group for three (six) quark flavours Soon afterwords the proposal of original quark model, the quark content of some hadrons was noticed to violate the Pauli's exclusion principle which consequently emerges as a problem of quark-statistics (*i.e.* the Fermi-Dirac statistics). In order to remove such apparent contradictions, an additional (hidden) DOF named colour [13] attached to each quark flavour was introduced which is essentially same as the introduction of colour quantum number [46, 47]. Despite various considerable efforts, the individual quarks with colour could not observed so far. However, the deep-inelastic lepton-nucleon scattering processes [48] have revealed strong dynamical evidences in favour of the existence of quarks and gluons (which mediates the strong colour forces between quarks) In particular in electron-nucleon scattering proceses, the scaling behaviour of the structure functions implies that the constituents of the nucleons look almost free and point-like particles known as partons [49,50] having same properties like quarks The presence of quark bound states have been, in fact, observed in e^+e^- annihilation processes which somehow get converted into a jet of hadrons (*i e* $e^+e^- \to q\bar{q} \to 2$ jets and e^+e $qqg \to 3$ jets) [51] These bound states in 2 and 3-jet events are represented in Figures 3 (a) and (b) respectively With such dynamical evidences for quarks and

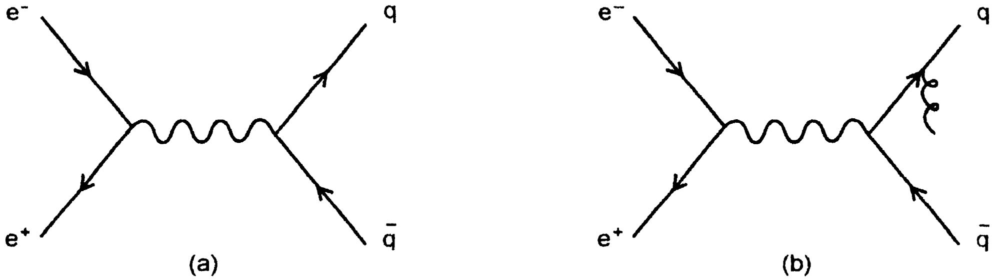


Figure 3 Hadrons in e^+e deep inelastic scattering experiments (a) 2 jet and (b) 3-jet events

gluons where the colour charges act as source of strong (hadronic) interactions, the various features of hadron physics have been synthesized with the development of the gauge field theory of strong interactions (*i e* QCD) The first step in the formulation of QCD is to identify the symmetry which is thought to be fundamental origin of strong (colour) forces The quark colour triplet (with red, blue, and green colour charges) under the symmetry group $SU(3)_C$ transformations generates the colour multiplets which categorise the hadrons These colour multiplets which represent the various combinations of colour for a given flavour of quark are completely distinct from the $SU(3)_F$ flavour multiplets The mediators of strong interactions between quarks in such formulation are the massless gauge bosons (gluons) which are self-interacting because of the non-Abelian nature of the colour gauge group $SU(3)_C$ On the other hand, such self-interaction among the gauge bosons of QED which are photons, is absent as the symmetry group $U(1)$ is Abelian in nature Thus, there is a possibility of the particles (glueballs) consisting of gluons bound together which are yet to be experimentally detected However, the colour electric charge of the strong interactions in QCD are analogous to the electric charges in electromagnetic interactions in QED A comparative view of the developments of the QED and QCD has been presented in Table 1 Moreover, by the same time of the formulation of QCD, a real breakthrough appeared in the gauge field theory of the strong interactions with the discovery of the asymptotic freedom which predicts that the intrinsic strength of the colour force between quarks decreases as they approach each other [53,54]. However, the full description of QCD

Table 1. A qualitative comparison of the structures of QED and QCD.

Theory	Gauge symmetry	Conserved charge	Coupling constant	Gauge bosons
QED	$U(1)$	Electric charge	$\alpha_{em} = e^2/4\pi$	One photon
QCD	$SU(3)_C$	Colour charge	$\alpha_s = g^2/4\pi$	Eight gluons

requires the formulation of the Lagrangian which is to be locally gauge invariant under the application of the $SU(3)_C$ gauge transformations which is outlined in the next section.

3.2. QCD as the gauge theory of strong interactions :

In order to gauge the colour symmetry of the strong interactions, the QCD is described by the $SU(3)_C$ non-Abelian gauge theory which provides a viable gauge field theory of strong interactions. The colour charges as the elements of the fundamental representation of $SU(3)_C$ colour gauge group (which is an unbroken gauge symmetry) along with the requirement of the renormalisability, therefore, fixes the QCD Lagrangian [9] in the following form,

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \sum_{j=1}^{N_F} \bar{\psi}_j \left(i\gamma^\mu D_\mu - m_j \right) \psi_j, \tag{24}$$

where, $a = 1, 2, \ldots\ldots 8$, N_F represents the number of quark flavours and m_j is the mass of the j-th quark. The covariant derivative D_μ and the field strength tensor $G^a_{\mu\nu}$ are given as,

$$D_\mu \psi_j = (\partial_\mu - igT_a W^a_\mu)\psi_j, \tag{25}$$

$$G^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - igf^{abc} W^b_\mu W^c_\nu, \tag{26}$$

where, W^a_μ represents the gluon fields while f^{abc} are the $SU(3)_C$ structure constants. The colour quark states transform under the fundamental representation of $SU(3)_C$ group which has eight generators as $T_a = \lambda_a/2$ with λ_a (3 × 3 traceless hermitian matrices known as the Gell-Mann matrices), and satisfy the commutation relations $[T_a, T_b] = if_{abc}T^c$. The Gell-Mann matrices with the normalisation condition $\mathrm{Tr}(\lambda_a, \lambda_b) = 2\delta_{ab}$ then satisfy the commutation relation $[\lambda_a, \lambda_b] = 2if_{abc}\lambda_c$. The structure constants f_{abc} appearing in these commutations relations are totally antisymmetric in their respective indices. In fact, the colour coding of quarks may be different at each point in space which requires the Lagrangian to be invariant under $SU(3)_C$ group of transformations. For this purpose, let us define a local $SU(3)_C$ gauge transformation in the following form,

$$U(x) = \exp\left\{ i\,\epsilon_a(x) T^a \right\}, \tag{27}$$

where, $\epsilon_a(x)$ are some arbitrary real functions. With the eq. (27), the quark and gauge fields then lead to the different transformations as follows,

$$\psi_j(x) \rightarrow \psi_j'(x) = U(x)\psi_j(x), \tag{28}$$

$$\boldsymbol{W}_\mu(x) = T_a W_\mu^a(x) \rightarrow \boldsymbol{W}_\mu' = U(x)\left\{\boldsymbol{W}_\mu(x) - ig^{-1}\partial_\mu\right\}U^{-1}(x), \tag{29}$$

$$\boldsymbol{G}_{\mu\nu}(x) = T_a G_{\mu\nu}^a(x) \rightarrow \boldsymbol{G}_{\mu\nu}' = U(x)\boldsymbol{G}_{\mu\nu}(x)U^{-1}(x) \tag{30}$$

The Lagrangian given by eq. (24) is invariant under the gauge transformations given by eqs (28–30) Such gauge invariance redefines the quark colour fields independently at every point in space-time without changing the physical spectrum of the theory Moreover, the gauge field strength tensor in such QCD Lagrangian (24) on expansion gives the self-interaction among the gauge fields (gluons) and leads to various interesting features in QCD those can be seen through the behaviour of strong coupling constant at different scales.

3.3. Running coupling constant and asymptotic freedom :

The dynamical effects of QCD are due to the self-interaction of gluons and it turns out to be extremely difficult to calculate the physical observables due to the appearance of the divergences in the theory These divergences need to be subtracted which is achieved by the renormalisation of the theory [62,63] involving a renormalisation scale μ. The renormalised coupling constant may then be obtained by defining the β-function [32] in the following form,

$$\beta(g) \equiv \mu\frac{\partial g}{\partial\mu}, \tag{31}$$

where g is the coupling parameter. The eq (31) takes the following value as calculated by using the perturbation technique,

$$\beta(g) \equiv \mu\frac{\partial g}{\partial\mu} = -\frac{g^3}{16\pi^2}\left(11 - 2\frac{N_F}{3}\right) = -bg^3. \tag{32}$$

In fact, with the change in the momentum scale, the effective gauge coupling $\bar{g} = (g,t)$ then obeys $d\bar{g}/dt = -b\bar{g}^3$, where, $t = \ln\mu$ which after integration leads to $1/\bar{g}^2 = 1/g^2 + 2bt$. Now, by choosing the scale parameter μ as the ratio of momentum of interest q to the subtraction scale q_0 *i.e.* $\mu^2 = q^2/q_0^2$, the coupling constant may then be written in the following form,

$$\alpha_s\left(q^2\right) = \frac{\alpha_s\left(q_0^2\right)}{1 + 4\pi b\alpha_s\left(q_0^2\right)\ln\left(q^2/q_0^2\right)}, \tag{33}$$

where, $\alpha_s(q_0^2) = g^2/4\pi$ and $\alpha_s(q^2) = \bar{g}^2/4\pi$. The eq. (33) may further be simplified by defining the parameter Λ_{QCD} as follows,

$$\Lambda_{QCD}^2 = q_0^2 \exp\left[-1/4\pi\, b\alpha_s\left(q_0^2\right)\right], \tag{34}$$

which consequently leads to,

$$\alpha_s\left(q^2\right) = \frac{4\pi}{\left(11 - 2\frac{N_F}{3}\right)\ln\left(q^2/\Lambda_{QCD}^2\right)}. \tag{35}$$

The strong coupling constant given by eq. (35) is presented in Figure 4. The Λ_{QCD} in eq. (35) plays a role of fundamental scale of QCD for the measurement of energy and known as the QCD scale parameter. It is clear from eq. (35) that $\alpha_s(q^2)$ increases for the small momenta while diverges at $q^2 = \Lambda_{QCD}^2$. The strong coupling constant (35) is obtained perturbatively which, therefore, breaks down for the large couplings in the deep infrared sector of QCD where various non-perturbative effects take place. On the other hand, the strong coupling constant which depends on b as in eq. (32), clearly implies that $\beta < 0$ as long as $N_F \leq 16$ so that the theory is asymptotically free at the high energy scales, where the mutual colour interactions of quarks and gluons can be neglected. It is important to notice that the scaling behaviour of effective coupling constant is given by the negative β-function which explains the asymptotic behaviour of the coupling strength [53,54]. However, in QED, the coupling constant $\alpha_{em}(q^2)$ decreases at small q^2 and shows a reversal of the picture of $\alpha_s(q^2)$ at small q^2 in QCD. Such behaviour of $\alpha_s(q^2)$, therefore, leads to the anti-screening process and the property of asymptotic freedom in QCD [53,54]. With these facts in mind, we will now try to discuss some basic aspects of dual QCD in the forthcoming sections and will first start with the techniques those Abelianise the QCD with the appearance of the magnetic charges in the next section.

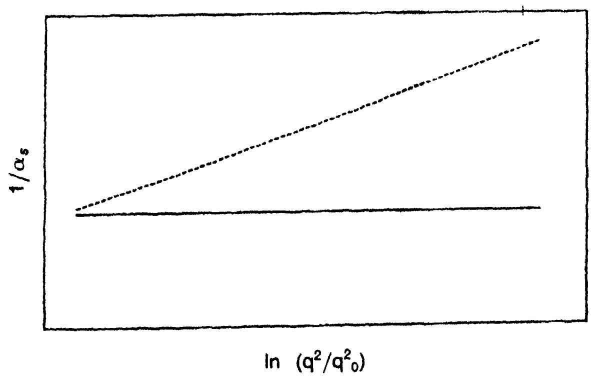


Figure 4. The strong coupling constant on logarithmic scale.

3.4. QCD as an Abelian gauge theory and monopoles :

The gauge fixing techniques which reduce the non-Abelian structure of QCD down to $U(1)$ (Abelian) theory with the magnetic sources are highly relevant because of the correspondence of QCD vacuum to the superconductivity and some of these techniques are discussed here. The idea of Abelianisation of QCD, indeed, starts with the 't Hooft's Abelian gauge fixing which is discussed next.

(i) 't Hooft's Abelian gauge fixing :

The '*t* Hooft's proposal of a particular unitary gauge fixing popularly known as Abelian gauge fixing [29] which reduces QCD to an Abelian gauge theory with the monopoles is quite remarkable. The concept of Abelian projection in to a non-Abelian gauge theory enables us to describe the physical variables of a non-Abelian gauge theory in terms of a set of electric and magnetic charge which interact *via* a residual Abelian gauge coupling. The choice of the gauge is chosen such that it reduces the gauge symmetry of the non-Abelian gauge group G to its maximal Abelian subgroup H. However the choice of the gauge is not unique. The Abelian gauge fixing is defined by the diagonalisation of a suitable gauge dependent variable $\boldsymbol{X}(x)$ which lies in the adjoint representation of G as follows,

$$\boldsymbol{X}(x) \to \boldsymbol{X}'(x) = \Omega(x)\boldsymbol{X}(x)\Omega^{-1}(x), \tag{36}$$

where $\Omega(x) \in G \equiv SU(N_C)$ is a gauge function *i.e.* $\Omega(x)$ are the elements of $SU(N_C)$. In fact, X is related with the Abelian gauge fixing as well as the gauge independent objects producing the crucial degrees of freedom and as the eigen values of X are gauge invariant, the most suitable choice of the gauge is one in which X is a diagonal *i.e.*,

$$X \equiv \mathrm{diag}\left(\lambda_1(x), \lambda_2(x), \cdots, \lambda_{N_C}(x)\right). \tag{37}$$

The gauge in which X is diagonal is called as Abelian gauge. In particular, for the choice of gauge group $G \equiv SU(N_C)$, the gauge defined by this choice is not determined completely since λ_i do not coincide at generic points. Any diagonal gauge rotation (Ω) then satisfy the condition,

$$\Omega \equiv \mathrm{diag}\,(e^{i\phi_1}, e^{i\phi_2}, \cdots e^{i\phi_N});\ \left(\sum\nolimits_i \phi_i = 0\right), \tag{38}$$

which leaves X invariant. The gauge field corresponding to this gauge freedom is given by the largest Abelian subgroup $U(1)^{Nc-1}$. There are indeed the points in the space where the Abelian gauge fixing is not uniquely defined (or ill defined) which act as the source of the singularities (monopoles) as topological defects in the space [26].

Let us perform the above mentioned arguments with the simplest case of the non-Abelian gauge group *i.e.* $SU(2)$. The suitable choice for X in the adjoint representation of $SU(2)$, at a particular point, is given by,

$$X = \mathrm{diag}\left(\lambda_1(x), \lambda_2(x)\right) = \sum a_0 + a_k\tau_k, \tag{39}$$

where τ_k are the well-known Pauli matrices and the eigen values of the matrix (39) are gauge independent. The appearance of the point-like singularities requires three constraints ($a_1 = a_2 = a_3 = 0$) and such singularities are only possible when the largest Abelian subgroup of the gauge group left unbroken. For the present case of the gauge group $SU(2)$, the singularity point may then be identified as $\vec{\epsilon}(x_0) = 0$, and the gauge condition corresponds to rotating $\vec{\epsilon}$ is such that, $\epsilon_3 > 0$ and $\epsilon_1 = \epsilon_2 = 0$, leaving X invariant with respect to the rotation around the third-axis. The zero point of $\vec{\epsilon}$ at x_0, then behaves as a magnetic charge of strength g with respect to remaining $U(1)$ rotations. The existence of such monopoles is, in fact, the most important feature of the '*t* Hooft's Abelian gauge fixing. The generalisation of the $SU(2)$ case for an arbitrary N_C is trivial.

In general, with the Abelian gauge choice for the gauge group $SU(N_C)$, it reduces to the maximal Abelian subgroup as $SU(N_C) \rightarrow U(1)^{N_C-1}$ having $N_C - 1$ fold multiplicity enriched with the magnetic monopoles which appear from the hedgehog like configurtions as mentioned earlier in Figure 2(b). These configurations widely appear in particle physics (*e.g.* the '*t* Hooft Polyakov monopole). The condensation of such monopoles in the ground state of QCD vacuum is presumed to play an essential role in explaining the confinement mechanism of quarks and gluons in the dual formulation of QCD.

(ii) Magnetic gauge fixing :

The introduction of the topological objects (*viz.* monopoles and dyons) in QCD vacuum and the dual dynamics between the colour iso-charges (quarks) and topological charges (monopoles) can also be described by the magnetic symmetry structure of the non-Abelian gauge theories [55,56]. The magnetic symmetry is defined as an additional isometry (H) of the internal fiber space in terms of a scalar multiplet (described by a set of self-consistent Killing vector fields $\hat{m}$) belonging to the adjoint representation of the gauge group (G) [55,56]. The magnetic symmetry described by $\hat{m}$, such that H is Cartan's sub group of the gauge group G, restricts the potential (connection) associated to G. It then results the gauge potential in terms of the electric (A_μ) and magnetic (B_μ) potentials which are Abelian in nature as shown later. With such considerations, the gauge covariant Killing condition may then be expressed as,

$$D_\mu \hat{m} \equiv \left(\partial_\mu + g\boldsymbol{W}_\mu \times\right)\hat{m} = 0\,, \tag{40}$$

where, $\hat{m}$ is the magnetic multiplet normalised to unity and $\boldsymbol{W}_\mu$ is the gauge potential associated with G and the cross product refers to the internal group space. The condition given by eq. (40) imposes strong constraint on the potential and here we will consider the simplest case of $G \equiv SU(2)$ with its little group H as $U(1)$. The exact solution of the eq. (40) leads to the following gauge potential,

$$\boldsymbol{W}_\mu = A_\mu \hat{m} - g^{-1}\left(\hat{m} \times \partial_\mu \hat{m}\right), \tag{41}$$

where, $A_\mu = \hat{m}\cdot \boldsymbol{W}_\mu$ is the Abelian (electric) component of $\boldsymbol{W}_\mu$, while the second part on the right hand side is completely determined by the magnetic symmetry which is topological in origin. The multiplet $\hat{m}$ may thus be viewed to identify the homotopy of the mapping $\Pi_2(S^2)$ as $\hat{m}: S_R^2 \rightarrow S^2 = SU(2)/U(1)$ where, S_R^2 is the 2-dimensional sphere of 3-dimensional space and S^2 is the group coset space completely fixed by $\hat{m}$. The monopole configurations then emerge as topological objects inherently associated with the elements of the second homotopy group. The field strength tensor ($\boldsymbol{G}_{\mu\nu}$) corresponding to the gauge potential ($\boldsymbol{W}_\mu$) may then be given in the following form,

$$\boldsymbol{G}_{\mu\nu} = \boldsymbol{W}_{\nu,\mu} - \boldsymbol{W}_{\mu,\nu} - g\boldsymbol{W}_\mu \times \boldsymbol{W}_\nu = (F_{\mu\nu} + B_{\mu\nu})\hat{m}\,, \tag{42}$$

where, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $B_{\mu\nu} = -g^{-1}\hat{m}\cdot\left(\partial_\mu \hat{m}\times\partial_\nu \hat{m}\right) \equiv \partial_\mu B_\nu - \partial_\nu B_\mu$ which represent the electric and magnetic counterparts respectively. The topological properties of magnetic symmetry as an additional isometry thus express the dynamics of QCD explicitly at the level of the electric and magnetic potential [55] as evident from the eq. (42). The dual symmetric separation of the gauge fields may then be used to bring the topological structure of the theory in to the dynamics by using the magnetic gauge which can be obtained by rotating $\hat{m}$ to a prefixed space-time independent direction in iso-space [55]. Since the monopoles in such formalism appear as point like objects with the singular behaviour of the magnetic potential (B_μ) and to get rid of these undesirable features, one can introduce the dual magnetic potential ($\tilde{B}_\mu$) with the associated gauge field strength tensor and a complex scalar field (ϕ) simultaneously for the monopole field [55,56]. This formalism in the quenched approximation (*i.e.* the absence of the coloured electric sources), ultimately leads to a Ginzburg-Landau type Lagrangian which has widely been used to study the confining mechanism in dual QCD vacuum in recent years [56,57].

(iii) Monopoles and dyons in lattice QCD :

The study of the field theories on the lattice (*i.e.* the lattice gauge theory) [58] where the space-time continuum is discretised is well-known [30]. In recent years, the Abelian dominance in QCD for non-perturbative quantities (like Wilson or Polyakov loops) is much speculated by the lattice gauge simulation studies [30,31]. There are strong evidences for monopole dominance in confinement process in the lattice gluodynamics. The monopoles in the lattice QCD can be characterised by using the maximally Abelian (MA) gauge where these objects are regarded as the gluon field fluctuations which are the relevant DOF for the confinement process. The monopoles in such Abelian projected QCD having a $U(1)$ gauge symmetry play an alternative role of the off-diagonal gluons and are able to reproduce the nonperturbative features of QCD [59]. Such monopoles are the important contributors to the string tension in QCD which characterises the strength of the confinement. These monopoles have an important role in the lattice gauge path integral with their possible applications to the dual superconductor models of QCD. The monopoles in the Abelian sector of QCD on lattice can, in fact, be visualised with a large Abelian action [45]. The QCD on lattice with monopoles provides a unique way to quantify the qualitatively known confinement parameters of crucial interest and the numerical results from lattice QCD at different distance scales also confirms the crucial property of asymptotic freedom and confinement in QCD as discussed in Section 3.3.

Further, in lattice QCD, the computer simulations also indicate that the Abelian monopoles also contain the electric charge (*i.e.* they behave as Abelian dyons) [60]. Such dyons carry the fluctuating electric charge. The correlation of the electric and magnetic currents is well studied in $SU(2)$ gluodynamics in MA gauge [61] and the infrared properties of the QCD in the background of such formulations can be described by using the Abelian Higgs model (AHM) of QCD where the dyons get condensed [60]. The flux tube structure (as discussed in Section 4.2) can also be presented with a non-zero string tension in view of the lattice gluodynamics in presence of either of the monopole or dyon condensation.

The emerging evidences regarding the monopoles/dyons in lattice QCD [45] (where the vacuum behaves as a magnetic (dual) superconductor as discussed in the next section) are thus expected to provide much deeper insights to the quark confinement as a dual Meissner effect in a four dimensional QCD.

3.5. The dual Ginzburg-Landau type formulation of QCD :

The above mentioned considerations leads to the correct physical description of the monopoles and dyons in the dual formulation of the QCD vacuum and the general form of the dual QCD Lagrangian derived from the naive QCD Lagrangian may then be expressed in the following form,

$$\mathcal{L} = -\frac{1}{4}\tilde{F}_{\mu\nu}\;\tilde{F}^{\mu\nu} + \left(\mathcal{D}_{\mu}\phi\right)^{*}\left(\mathcal{D}^{\mu}\phi\right) + V(\phi)\,, \tag{43}$$

where, $\mathcal{D}_{\mu} \equiv \partial_{\mu} - ig\tilde{A}_{\mu}$ and $\tilde{F}_{\mu\nu} = \partial_{\mu}\tilde{A}_{\nu} - \partial_{\nu}\tilde{A}_{\mu}$ is the field strength tensor constructed on the dual gauge field $\tilde{A}_{\mu}$. The scalar field ϕ represents the monopole field and it has a non-zero magnetic charge g. The $V(\phi)$ is the effective potential which generates the mass of the dual gauge field in the broken phase of symmetry and consequently the features of magnetic superconductivity in the condensed mode of QCD vacuum. In fact, $V(\phi)$ ensures that $<\phi> \neq 0$ in vacuum and the monopole field plays the role of Ginzburg-Landau (GL) order parameter like the macroscopic Cooper-pair wave function in the conventional (electric) superconductivity. The mass acquisition of the dual gauge field in QCD can be clearly seen in terms of the following massive vector field equation associated to the Lagrangian given by eq. (43),

$$\left(\Box + m_A^2\right)\tilde{A}_\mu - \partial_\mu\left(\partial_\nu \tilde{A}^\nu\right) = ig\left\{\phi^*\partial_\mu\phi - \phi\partial_\mu\phi^*\right\}, \tag{44}$$

where, $m_A = g\phi_0$ is the mass of the dual gauge field. Since the complex scalar field has a finite value at each space-time point (*i.e.* $\partial_\mu\phi^* = 0 = \partial_\mu\phi$) and with an astute choice of gauge which is the Lorentz gauge, the eq. (44) reduces to a massive vector type equation. With such considerations, the simplest solution of eq. (44) may thus be derived in the half-space of all the space as being filled by the stochastic phase ($i.e.$ $\varphi_0 \neq 0$) for $x \geq 0$. The dual gauge field has then only a dependence on x and $\tilde{A}_\mu$ as follows,

$$\left(\partial_x^2 - m_{\tilde{A}}^2\right)\tilde{A}_\mu = 0, \tag{45}$$

which in turn results,

$$\tilde{A}_\mu = \tilde{A}_{0\mu}\exp\left(-m_{\tilde{A}}x\right), \tag{46}$$

where, $\tilde{A}_{0\mu}$ is a constant vector and the eq. (46) consequently guarantees a dual Meissner effect which can be seen more transparently at the level of colour electric field as discussed in the Section 4.1.

4. Dual QCD vacuum and flux tube structure

The elementary processes in QCD for small value of $\alpha_s(q^2)$ at large momenta [64] where the quarks and gluons become asymptotically free, can be calculated by using the perturbative techniques. On the other hands, at low momenta $\alpha_s(q^2)$ increases rapidly such that the usual perturbative expansions are not possible and the system faces a transition from perturbative to non-perturbative phase which is maintained by Λ_{QCD} having the value (220 ± 15 ± 50) MeV as predicted by $\overline{MS}$ schemes [65]. The large couplings in QCD are presumably connected with the confinement of quarks at large distances as all the observed hadrons are colour singlets with $q\bar{q}$ (mesons) and qqq (baryons) bound states. The indirect experimental evidence for these confined states comes from the hadron spectroscopy of the heavy quarkonia (*i.e.* Charmonium $J/\Psi = c\bar{c}$ and Bottomonium $\Upsilon = b\bar{b}$). Apart from the confinement of quarks, the QCD at large distances has also various other phenomena *viz.* the dynamical chiral symmetry breaking (DχSB) and bulk properties of QCD vacuum [66]. The correct non-perturbative description of these problems seems possible through dual superconductor models of QCD which is characterised by the monopole condensation [66]. Such a QCD vacuum has a nice dual analogue in the Higgs model and when the dual Abelian $U(1)$ gauge symmetry is spontaneously broken, the theory has the features of magnetic superconductivity which is discussed in the next section.

4.1. Analogy of QCD vacuum with superconductivity :

The dual superconducting model of QCD provides an explanation of confinement as the dual of the superconductivity. In order to discuss such scenario, let us first consider the Lagrangian with Higgs mechanism in scalar QED as follows,

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \left(D_\mu \varphi\right)^* \left(D^\mu \varphi\right) + \frac{\mu^2}{2} \varphi \phi^* - \frac{\lambda}{4} \left(\varphi \varphi^*\right)^2, \tag{47}$$

where, $D_\mu \equiv \partial_\mu - ieA_\mu$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. For static case (*i.e.* $\partial^0 \varphi = \partial^0 \boldsymbol{A} = 0$ and $A_0 = 0$), the field equation for $\boldsymbol{A}$ can be given in the form given below,

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} = ie\left\{\varphi^* \left(\nabla - ie\boldsymbol{A}\right)\varphi - \left(\nabla + ie\boldsymbol{A}\right)\varphi^* \varphi\right\}, \tag{48}$$

where, $\boldsymbol{H}$ is the magnetic field. In the spontaneously broken phase of symmetry, the current satisfy the following local relation which is known as the London equation,

$$\boldsymbol{J} = e^2 v^2 \boldsymbol{A}, \tag{49}$$

where, $v = \sqrt{\mu^2/\lambda}$. The eq. (49) in turn leads to,

$$\nabla^2 \boldsymbol{H} = e^2 v^2 \boldsymbol{H}, \tag{50}$$

where, we have considered $\nabla \cdot \boldsymbol{H} = 0$. The eq. (50) in one dimension in the half space $x \geq 0$ gives the magnetic field as,

$$\boldsymbol{H}(x) = \boldsymbol{H}(0) \exp(-x/\lambda), \tag{51}$$

where, $\lambda = m_A^{-1} = (ev)^{-1}$ is the penetration depth which is the inverse of the vector gauge field mass. The eq. (51) implies Meissner effect indicating that the magnetic field decay in a distance λ. The dual analogue of this scenario is also present in QCD and can be visualised through the Lagrangian for dual QCD in the quenched approximation which is given by eq. (43). There is a striking parallelism among the consequences drawn from the Lagrangian (43) and the above mentioned superconductivity as a Higgs phenomena in QED with Lagrangian (47). In this dual version of superconductivity, the dual vector field acquires the mass as a result of symmetry breakdown followed by the magnetic charge condensation.

In oder to visualise such analogy, let us consider the electric field $\tilde{\boldsymbol{E}} = \nabla \times \tilde{\boldsymbol{A}}$ for the case of dual QCD with Lagrangian (43) which in the static limit satisfy,

$$\nabla^2 \tilde{\boldsymbol{E}} = m_{\tilde{A}}^2 \tilde{\boldsymbol{E}}, \tag{52}$$

and for one dimensional case [56], it leads to,

$$\tilde{\boldsymbol{E}}(x) = \tilde{\boldsymbol{E}}(0) \exp(-m_{\tilde{A}} x). \tag{53}$$

The eq. (53) indicates that the electric field screened out a distance $\tilde{\lambda} = m_{\tilde{A}}^{-1}$ which is the penetration depth where $m_{\tilde{A}}$ is the dual gauge field mass. The eq. (53)

guarantees a dual Meissner effect and hence a flux tube structure between a quark and antiquark More generally, this dual Meissner effect prevents the colour electric flux to spread out and yield squeezing of the chromoelectric field in to a flux tube Further the coherence length ($\tilde{\xi}$) of the monopole condensate is defined as the inverse of the scalar field mass (*i e* m). These two length scales determine the superconducting nature of QCD vacuum *i e* whether is of type-I ($\tilde{\xi} > \tilde{\lambda}$) or of type-II ($\tilde{\xi} < \tilde{\lambda}$) These two length scales also leads to the dimensionless Ginzburg-Landau (GL) parameter $\kappa = \tilde{\lambda}/\xi$ which is useful in predicting the superconducting type of QCD vacuum [56] For the case, when $\kappa > 1$, the QCD vacuum is of type-II nature while for $\kappa < 1$ it behaves like a type-I superconductor. However, the equality of the length scales $\tilde{\xi}$ and $\tilde{\lambda}$ (*i e* $\kappa = 1$) gives a transition from type-I to type-II superconducting QCD vacuum Defining the type of superconductor may helpful to clarify the dynamics of colour confinement and the interaction of the flux tubes more precisely The flux tube model is indeed among the most fruitful ideas concerning the confinement mechanism which is discussed in the next section

4 2 The flux tube model and confinement

The superconducting picture of QCD, in fact, replaces the electric charge (electron) by chromomagnetic charge (monopole) and consequently the monopole-pairs (as the dual analogue of the Cooper-pairs in superconductivity) get condensed in QCD vacuum which give rise to the dual Meissner effect Indeed, the condensed QCD vacuum is somewhat analogous to the ground state of a superconductor The colour electric flux between chromoelectric sources (quarks) then squeezed into thin flux tubes and, thereby confining the quarks in a strongly localised region However, the phenomenological Cornell (funnel) potential between the quarks [68] having such flux tube structure is given in the following form,

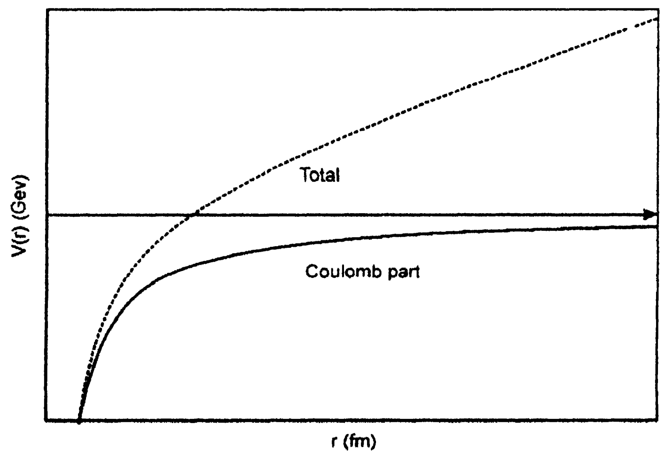


Figure 5. Schematic view of the Cornell potential at $\alpha_s \sim 0\,5$ and $\sigma \sim$ GeV/fm

$$V_{q\bar{q}}(r) = -\frac{4}{3}\frac{\alpha_s}{r} + \sigma r, \tag{54}$$

where, the first term in eq. (54) is the contribution to the potential due to one-gluon exchange (OGE) process at short distances and the coefficient 4/3 in the first term arises due to the summation over different colour components corresponding to the colour singlets in nature. The second term in eq. (54) which is linear in r indicates that the quarks experience a constant force at large distances. The density of the field lines of the flux tube structure is independent of r and the quarks are, therefore, absolutely confined because an arbitrarily large amount of energy is needed to separate them. The first term in eq. (54), however, dominates over the second term at sufficiently small r and the quark-pair then experiences only the Coulomb force. A qualitative view of the

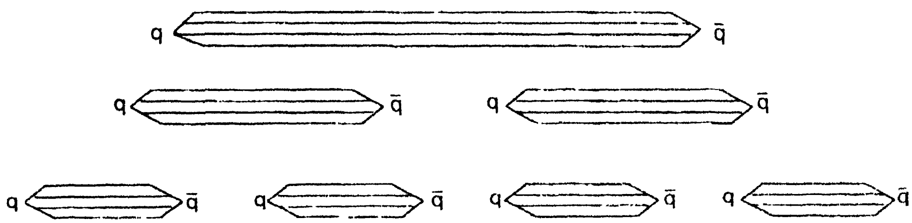


Figure 6. The flux tube structure and meson pair creation

phenomenological potential is presented in Figure 5. If the quark and anti-quark move apart, the breaking of the flux tube is accompanied by the quark-pair production (*i.e.* mesons) as shown in Figure 6. This situation is somewhat like a perfectly elastic band which on sufficient stretching breaks into two pieces. Moreover, the evidence of such linearly rising distance potential at large distances as given in eq. (54) also comes through the linearly rising trajectories of hadrons. In fact, the light hadrons (*i.e.* the particles consisting from light u, d and s quarks) are well described by the linear Regge trajectories and the observed relation between the angular momentum quantum number and the energy of a hadron state is given by,

$$J = \alpha' M^2 + \alpha_0, \tag{55}$$

where, $\alpha_0 \sim 0.5$ is a constant and α' is Regge slope parameter (RSP) is given by $\alpha' = (2\pi\sigma)^{-1}$ with σ as the linear energy density per unit length of the flux tube (*i.e.* string tension). The eq. (55) holds for the case of constant energy density σ of the flux tube [69], that is, to a potential of the form $V_{q\bar{q}}(r) = \sigma r$ where, r is the inter-quark distance. This linear potential is also necessary to ensure the correct ordering of the energy levels in the observed energy spectra of J/Ψ and Υ bound states. The phenomenological potential (54) is thus trustworthy in view of the hadron spectroscopy of the heavy quarkonia (*i.e.* when their masses are large in comparison to the Λ_{QCD}) In such potential, the string tension is the only phenomenological parameter which decides the strength of the confinement. The linearly rising Regge trajectories are, therefore, relevant to the linear form of QCD potential and confirm the validity of the flux tube model of hadrons. The observed value of RSP which is ($\alpha' \simeq 0.93$ GeV $^{-2}$) then leads $\sigma \simeq 0.87$ GeV/fm and the same result also comes from the consideration of the measured size of the hadrons in electron scattering experiments [69]. In general $\sigma \sim 1$ GeV/fm which is the key ingredient of confinement and we have graphically presented the total phenomenological potential and its Coulombian part separately. The nature of confinement potential is also extensively studied in the background of magnetic condensation and it is shown to be composed of the Yukawa counterpart accompanied by a linear potential. The potential depends on the mass scales of a particular model *i.e.* on the dual gauge and scalar field masses. For OGE process, the Yukawa part reduces to usual Coulombian one as in eq. (54). It is also worth mentioning that the RSP for the case of dyon condensation changes slightly in comparison to the case of pure monopole condensation. This is because of the enhancement in the string tension of the flux tube which is due to the electric excitation of the monopoles. However the linearly rising behaviour of Regge trajectories and confinement potential remain intact [70].

5. Conclusions

The monopoles and dyons are emerged as topological objects when a non-Abelian gauge theory like QCD is reduced to an Abelian one through a particular gauge fixing. As such with the existence of monopoles in QCD, their condensation in the broken phase of symmetry leads to the dual Meissner effect which constricts the colour electric flux in to thin tubes and paves the way for quark confinement in dual QCD. In recent years, the dual description of QCD has, in fact, played an important role in explaining various low energy properties of hadrons by incorporating a direct analogy of the nature of QCD vacuum with the conventional superconductivity. The firm link of confinement mechanism to the magnetic condensation is also tested with convincing arguments in the lattice QCD with the advanced modern age computation techniques. It seems that the confinement mechanism and the monopole/dyon condensation are inextricably locked with each other and one process can not be evoked without other. Moreover, the dual QCD can also become a testing laboratory for the monopoles those remain elusive at the experimental level so far. The dual superconductor models in QCD along with their motivation from lattice QCD are, therefore, quite pertinent and useful in defining the intrinsic properties of the hadrons and it is believed that they may helpful to explain various non-perturbative aspects of QCD like the DχSB and dielectric nature of QCD vacuum along with the long standing confinement problem. However, the precise behaviour of inter-quark forces at the level of quarks and gluons is still one of the centres of attention for particle physicists.

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About the Authors

H Nandan obtained Ph.D degree from Kumaun University, Nainital in 2006. He has worked on the dual superconductor models of QCD and has published 5 papers in this area. Presently, he is working as a Research Associate at the Centre for Theoretical Studies (CTS), Indian Institute of Technology, Kharagpur. His current research interests include the Raychaudhury equations, kinematics of flows and scalar-tensor theory of gravity with Higgs field. He has also participated in various schools, workshops and conferences related to his field of research interests.

H C Chandola is a senior Reader at the Department of Physics, Kumaun University, Nainital and working in the field of Theoretical High Energy Physics since last more than 25 years. Major fields of his interests are QCD, QGP, topological aspects of gauge theories, Special Relativity, Non-Abelian gauge theories and unification of fundamental forces. He has published more than 70 papers in journals of international repute. He has also awarded international fellowships by Indo-German Academic Exchange Service (Germany) and Commission of European Communities. He has participated in more than 20 Seminars/conferences/schools in India and abroad.